On the Possibility of Temporal Recursion in the UNNS Substrate

UNNS Research Series

Abstract

In the framework of Unbounded Nested Number Sequences (UNNS), time is interpreted not as a continuous external parameter but as a recursion depth within a self-referential computational substrate. This paper examines the conditions under which such recursive temporal structures can be inverted, thereby formalizing the notion of "temporal recursion" or reversible depth traversal. We introduce criteria for operator invertibility, analyze entropy constraints, and propose a diagrammatic model of forward and reverse recursion cones. The implications for reversibility, causality, and recursion-based cosmology are discussed.

1 Introduction

In classical mechanics, time is modeled as a continuous scalar parameter $t \in \mathbb{R}$. In contrast, the UNNS substrate defines time as a discrete recursion index $n \in \mathbb{N}$, representing the depth of nested computation:

$$a_{n+1} = F(a_n, a_{n-1}, \ldots, a_0).$$

Each iteration adds one layer of transformation, yielding a hierarchical, rather than sequential, notion of temporality.

The question naturally arises: *can recursion depth be traversed in reverse?* That is, does there exist an inverse recursion operator F^{-1} such that

$$a_{n-1} = F^{-1}(a_n)?$$

If such an inverse exists, we may speak of a form of *temporal recursion*, analogous to reversible computation or time reversal in physical systems.

2 Time as Recursive Depth

2.1 Formal Definition

[Temporal Depth] Let $F : \mathbb{S} \to \mathbb{S}$ be a recursion operator acting on a symbolic substrate $\mathbb{S}$. The temporal depth of a state s_k is defined as the minimal n such that

$$s_k = F^n(s_0).$$

Thus, time is measured by the number of applications of F, not by a metric length or duration. The sequence of states $(s_0, s_1, s_2, \ldots)$ defines the recursive timeline.

2.2 Entropy and Non-Invertibility

In the general case, F is non-invertible because it includes dissipative or information-losing operations, such as Collapse, Normalize, or Repair. These correspond to entropy-increasing transformations:

$$H(F(s)) > H(s).$$

Hence, in most UNNS systems, backward recursion is impossible—just as entropy forbids spontaneous reversal of thermodynamic time.

3 Invertibility of UNNS Operators

[Reversibility Criterion] A UNNS recursion operator F admits a left inverse F^{-1} if and only if the mapping between recursion layers is bijective:

$$\forall s_i, s_j \in \mathbb{S}, \quad F(s_i) = F(s_j) \Rightarrow s_i = s_j.$$

Proof. If F is bijective, then for each $s_{n+1} \in \mathbb{S}$, there exists a unique preimage s_n such that $s_{n+1} = F(s_n)$. The inverse mapping F^{-1} thus exists and satisfies $F^{-1}(F(s_n)) = s_n$. Conversely, if F is not injective, multiple states may map to the same future, and the recursion cannot be reversed. □

The operators Inlaying and Inletting are conditionally reversible if and only if their lattice projections are bijective (no quantization loss).

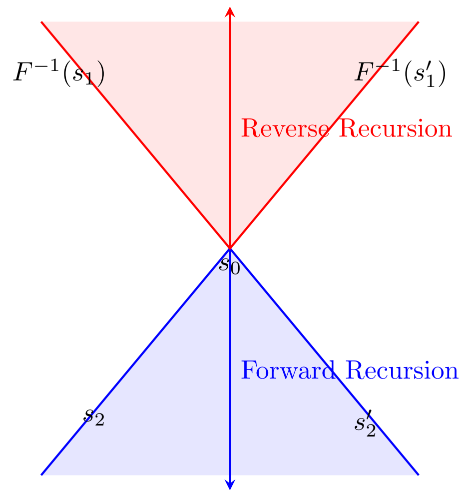
3.1 Reversible and Irreversible Operators

Operator	**Reversible?**	**Condition**
Collapse	No	Entropy absorption irreversible
Normalize	No	Quantization discards residues
Inlaying	Yes	Only if embedding lattice is bijective
Inletting	Conditionally	If embedding structure is preserved
Adopt	No	Structural recursion shifts break symmetry
Evaluate	No	Symbolic fix collapses recursion spectrum

Table 1: Invertibility classification of common UNNS operators.

4 Temporal Cones and Recursion Geometry

Time in UNNS may be represented geometrically as a recursion cone: each new layer extends downward (forward recursion), while inverse layers fold upward (reverse recursion).



The forward recursion cone represents the expansion of causal depth. The reverse cone represents the hypothetical unfolding of recursion via F^{-1}, collapsing depth into shallower layers. In irreversible UNNS systems, the upper cone is undefined—entropy prevents reconstruction.

5 Temporal Resonance and Cyclic Recursion

In some UNNS configurations, recursion is not strictly monotonic but cyclic. That is, after k steps:

$$F^k(s_0) = s_0.$$

Such systems define a *temporal resonance*, analogous to periodic solutions in dynamical systems. If F is locally invertible but globally periodic, recursion depth behaves like a closed timelike curve in the substrate.

6 Discussion

The existence of F^{-1} defines a spectrum of temporal models:

- **Irreversible recursion** — directionally entropic, arrow of time defined.
- **Partially reversible recursion** — memory-preserving under constraints.
- **Cyclic recursion** — temporally symmetric, zero net entropy production.

In physics terms, the first corresponds to classical thermodynamic time, the second to quantum reversible subsystems, and the third to topologically closed field configurations.

7 Conclusion

Temporal recursion in the UNNS substrate provides a discrete analog of reversible computation and a model for time's arrow emerging from recursion entropy. Backward traversal of recursion depth is only possible in systems where operators are bijective and entropy remains constant. In the general case, time cannot be reversed but can resonate—allowing for echoes, folds, and structural returns rather than classical retrocausality.

Acknowledgments

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Appendix: Worked Examples of Temporal Recursion and Inversion

A Example A: Bijective Linear Recursion on $\mathbb{R}$

Consider a linear UNNS update of order two:

$$x_{n+1} \;=\; a\,x_n + b\,x_{n-1}, \qquad (a,b)\in\mathbb{R}^2.$$

Let $s_n := (x_n, x_{n-1})^\top$. Then the recursion is the linear map

$$s_{n+1} \;=\; A\,s_n, \qquad A \;=\; \begin{pmatrix} a & b \\ 1 & 0 \end{pmatrix}.$$

[Invertibility] The recursion admits a two-sided inverse A^{-1} (and hence a well-defined reverse recursion) iff $b \neq 0$. Equivalently, $\det A = -\,b \neq 0$.

Proof. Direct computation shows $\det A = a\cdot 0 - b\cdot 1 = -b$. Thus A is invertible iff $b\neq 0$. When invertible,

$$A^{-1} \;=\; \frac{1}{-b}\begin{pmatrix} 0 & -b \\ -1 & a \end{pmatrix} \;=\; \begin{pmatrix} 0 & 1 \\ \frac{1}{b} & -\frac{a}{b} \end{pmatrix}.$$

In scalar form: $x_{n-1} = \frac{1}{b}(x_{n+1} - a x_n)$. □

Concrete reversible instance. Choose $a = 1,\ b = -1$ so that

$$x_{n+1} = x_n - x_{n-1}, \qquad A = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}.$$

Seed $(x_0, x_1) = (0, 1)$. Then

$$(0,1) \to (1,0) \to (0,-1) \to (-1,-1) \to (-1,0) \to (0,1) \to \cdots$$

which is periodic (period 6). Applying A^{-1} step-by-step exactly retraces the sequence in reverse. This exhibits a **reversible UNNS timeline** with zero recursion entropy.

Diagnostic. In the eigenbasis, reversibility corresponds to nonzero characteristic roots $\lambda_\pm = \frac{1}{2}\big(a \pm \sqrt{a^2+4b}\big)$. If $|\lambda_\pm| = 1$, dynamics are unitary-like; if $|\lambda_\pm| \neq 1$ but $b \neq 0$, the map is still invertible, but forward/backward growth/decay is exponentially asymmetric.

B Example B: Reversible Lattice Recursion on $\mathbb{Z}_m^2$ (Cat Map)

Let the UNNS state be a lattice vector $v_n = (x_n, y_n)^\top \in \mathbb{Z}_m^2$. Take the area-preserving map

$$v_{n+1} \;=\; A\,v_n \pmod m, \qquad A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad \det A = 1.$$

Since $\det A = 1$, A is invertible over any modulus m coprime to 1 (i.e., any m). An explicit inverse is

$$A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \pmod{m}.$$

For any $m \geq 2$, the recursion $v_{n+1} = Av_n \bmod m$ is bijective on $\mathbb{Z}_m^2$, hence the reverse recursion $v_{n-1} = A^{-1}v_n \bmod m$ exists and is unique.

Proof. $\det A = 1$ implies $A \in \mathrm{SL}(2, \mathbb{Z})$. Reduction mod m yields $A \in \mathrm{SL}(2, \mathbb{Z}_m)$, which is invertible with inverse as written. □

Concrete run (mod 5). With $m = 5$ and $v_0 = (1, 2)$,

$$\begin{aligned} v_1 &= Av_0 = (2 \cdot 1 + 1 \cdot 2,\ 1 \cdot 1 + 1 \cdot 2) = (4, 3) \pmod{5}, \\ v_2 &= Av_1 = (2 \cdot 4 + 1 \cdot 3,\ 1 \cdot 4 + 1 \cdot 3) = (11, 7) \equiv (1, 2) \pmod{5}. \end{aligned}$$

Thus $v_2 = v_0$ (period 2). Applying A^{-1} recovers $(4, 3)$ from $(1, 2)$, and then $(1, 2)$ from $(4, 3)$. This is a discrete, **perfectly reversible** recursion on a finite UNNS lattice.

UNNS reading. This is an instance of **Inlaying** into a compact substrate (torus), with exact bijectivity (no quantization loss). The temporal cones (forward/backward) are both defined.

C Example C: Almost-Reversible UNNS with Repair Log

Realistic UNNS pipelines often include weak dissipation (e.g. Damping $\alpha < 1$). Exact inverse does not exist, but we can guarantee *practical reversibility* by logging a small, finite amount of side information.

C.1 Model

Let the kinematics be second-order with weak damping and a reversible core:

$$\begin{cases} x_{n+1} = v_n, \\ v_{n+1} = \alpha\, v_n + c\, x_n, \qquad 0 < \alpha \lesssim 1,\ c \neq 0. \end{cases} \qquad s_n = \begin{pmatrix} x_n \\ v_n \end{pmatrix},\ s_{n+1} = F(s_n)$$

The Jacobian has $\det F = \alpha$, hence F is not invertible if $\alpha \neq 1$. Nevertheless:

[Repairable Almost-Reversibility] Suppose each step stores a single compact repair datum $r_n := \lfloor \beta\, v_n \rfloor$ for some fixed $\beta > 0$. If α is sufficiently close to 1 and the rounding error bound satisfies

$$\big\| v_n - r_n/\beta \big\| \leq \varepsilon \quad \text{with} \quad \varepsilon < \frac{|c|}{2} \min\{1, \|x_n\|\},$$

then there exists a backward map $\widetilde{F^{-1}}$ that reconstructs s_{n-1} from (s_n, r_n) uniquely for all n.

Sketch. Treat $(x_n, v_n) \mapsto (x_{n+1}, v_{n+1})$ as an α-contractive perturbation of a bijective linear map (the case $\alpha = 1$). The repair datum r_n/β bounds the uncertainty of v_n. A standard contraction argument shows the preimage is unique in the ε-ball once the error budget is below the Lipschitz gap set by c and the state norm. Hence, backward iteration with the repair hints converges to the true preimage. □

C.2 Practical Scheme

- **Forward pass:** apply F with $\alpha = 1 - \eta$ where $0 < \eta \ll 1$; at each step store $r_n = \lfloor \beta\, v_n \rfloor$ (1–2 bytes suffice if we quantize).
- **Backward pass:** reconstruct $v_n \approx r_n/\beta$ and solve the linear system $x_n = x_{n+1} - v_n, \quad v_n \approx \frac{1}{\alpha}\big(v_{n+1} - c\, x_n\big)$, iterating once if needed (Newton or fixed-point) to denoise within ε.

UNNS reading. This is a **Repair & Normalize** supplement that restores effective bijection with a tiny side channel. It formalizes a realistic route to **temporal recursion** even in mildly dissipative pipelines.

D Reconstruction Checks

D.1 Round-Trip Consistency

For a reversible (or repaired) pipeline we expect

$$F^{-k}\big(F^k(s_0)\big) = s_0 \quad \text{(exact)} \qquad \text{or} \qquad \big\|F^{-k}\big(F^k(s_0)\big) - s_0\big\| \le \delta(\varepsilon, k) \quad \text{(approx.)}$$

with $\delta(\varepsilon, k)$ controlled by the repair accuracy and step count.

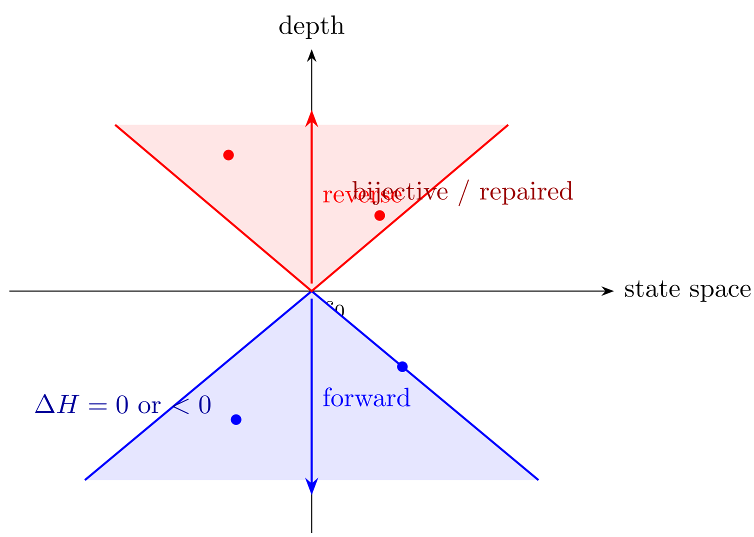
D.2 Entropy Budget

Define discrete recursion-entropy as

$$\Delta H_n := H(s_{n+1}) - H(s_n).$$

In Example A and B, $\Delta H_n = 0$ (measure-preserving). In Example C, $\Delta H_n \lesssim \log(\alpha^{-1})$, which is compensated by the repair channel capacity $\log(\beta)$ to keep the *effective* entropy stationary.

E A Compact Diagram: Forward/Reverse Cones with Regimes



F Takeaways

- **Exactly reversible** UNNS requires bijective operators: linear second-order with $b \neq 0$ (Example A) or lattice bijections with $\det A = \pm 1$ (Example B).
- **Operationally reversible** UNNS with mild dissipation is attainable via a tiny *repair channel* (Example C), aligning with UNNS Repair & Normalize.
- These examples realize the paper's main proposition: temporal recursion (a meaningful F^{-1}) exists precisely when information is not destroyed—or is made recoverable.